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Oscillations of Faster than Light Majorana Neutrinos: A Causal Field Theory¹

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Abstract

In this paper, we carry out the canonical quantization of the field theory of an interacting tachyonic Majorana neutrino. We show how micro-causality is preserved in the physical scattering matrix elements between the in and out vacua.

The phenomenology of this radical proposal is nevertheless compatible with normal timelike oscillations.

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1 Introduction

Recent data from Super-Kamiokande[1] suggest that neutrinos oscillate and are therefore massive. But does this mean that they are necessarily Dirac neutrinos, or can they be Majorana neutrinos ?

There has been much recent interest on this possibility[2]. In this paper, we propose an observation that the neutrino can not only be a Majorana particle, but, in an interesting twist, has a tachyonic[3] mass. (See ref[4] for earlier work on a tachyonic neutrino.) We shall show that the phenomenology of such a radical proposal is nevertheless compatible with normal timelike oscillations.

Problems with Tachyon Field

Earlier studies of tachyon field theory encountered many difficulties. It is well-known that a tachyon field ψ_{sp} containing only spacelike superluminal modes ($k > m$) does not obey micro-causality. The equal time anti-commutator $\{\psi_{sp}(\vec{x}, 0), \psi_{sp}^\dagger(\vec{y}, 0)\}$ is not a simple spatial delta function, but has a space-like tail that destroys micro-causality.

If in trying to solve the micro-causality problem, you include modes with $k < m$, you encounter complex solutions, and the problem becomes one of physical interpretation of exponential runaway states.

Majorana Field Theory

In this paper, we study the quantum field theory of a pair of opposite metric massless left-handed fields (see eq.(9)) whose Majorana interactions lead to tachyonic modes for the physical matrix elements. We

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carry through the complete canonical quantization in order to clarify the issues of superluminal propagation and its impact on micro-causality. We find that, in addition to the expected Lorentz invariant space-like measure

$$\int d^4 p \delta(p \cdot p - m^2) \quad (1)$$

for the neutrino field, there is a missing Lorentz invariant measure that must be included in the field. It is a measure for complex momenta

$$\int d^4 q \delta(q \cdot q - m^2) \theta(m^2 - q \cdot \bar{q}) \quad (2)$$

Here, we are using the metric such that

$$q \cdot q = \vec{q} \cdot \vec{q} - q_o \cdot q_o \quad (3)$$

$$q \cdot \bar{q} = \vec{q} \cdot \vec{q}^* - q_o \cdot q_o^* \quad (4)$$

The integration contour for $d^3 q$ can, for the space of holomorphic functions, always be brought to along the real axes, so that the complex measure becomes an integration over imaginary q_o . The field operator expansion thus contains both the exponential decaying and run-away modes, and appears at first sight to cause difficulties with large time limit ($t \rightarrow \pm\infty$) as well as with time translation invariance of the operator product matrix elements. Our study shows the remarkable property, however, that the run-away modes decouple from the physical matrix elements and only the *transient* exponential decaying modes remain. Furthermore, space-time translation invariance is preserved.

This comes about through the two nilpotent vacua in this theory, which can be shown to be the *in* and *out* vacua of the scattering matrix.

Physical vacuum matrix elements

The physical matrix element of the equal time anti-commutator taken between the *in* and *out* vacua remains a simple delta function,

$$(\Phi_{out} | \{ \psi_L(\vec{x}, t), \psi_L^\dagger(\vec{y}, t) \} | \Phi_{in}) = \delta(\vec{x} - \vec{y}) \quad (5)$$

while the corresponding vacuum expectation value of the ψ_L field satisfies the causality condition

$$(\Phi_{out} | \{ \psi_L(x), \bar{\psi}_L(y) \} | \Phi_{in}) = 0 \quad \text{for } (x - y)^2 \text{ spacelike} \quad (6)$$

As a result of this causal property, the S-matrix operator continues to be given by the time-ordered evolutionary operator even in the presence of propagation of tachyonic neutrinos.

The time-ordered Green function for the neutrino propagation from space-like separated x to a later y , ($y_o > x_o$), when viewed in a different frame with x'_o later than y'_o , is according to eq.(6) actually the time-ordered Green function for the antineutrino propagation from y' to x' . The physical matrix elements are correctly described by retarded Green functions, so that there are no problems arising from the acausal propagation of neutrino backwards in time.

Nilpotency of the physical vacua

The nilpotency results from the Majorana coupling between the physical ψ_L field and the sterile negative metric ψ'_L field. This ghost field does not couple to the strong, electromagnetic or weak interactions. It does not occur in the physical S-matrix elements. Its only function is to condense with the physical ψ_L field to produce an ether (the *in* and *out* vacua) that support quasi-particle modes with $v > c$ as well as the transient modes with $k < m$.

In this picture, the vacuum is itself a neutrino condensate so that in a sense the neutrino's role in our universe transcends that of the photon. While light illuminates the universe, and is the dominant signal

carrier from one part of the universe to another, the low energy neutrino is the weak and largely silent and transparent *superluminal* courier that pervades throughout the universe.

This model has interesting implications for the phenomenology of neutrino oscillation. In the one flavor case, the physical neutrino state (with $p > m$) created at time $t = 0$ propagates with no neutrino deficit. But in a multiflavor generalization, the physical neutrino states oscillate as a result of the mixing between the flavor fields and the mass eigenstates. The resulting neutrino oscillation phenomenology is indistinguishable from the usual Dirac flavor mixing oscillation.

2 Majorana Equation

Consider the following coupled set of Majorana field equations for the mutually anticommuting left-handed fermion field operators, $\psi_L(x, t)$, $\psi'_L(x, t)$:

$$(\vec{\gamma} \cdot \vec{\nabla} - \gamma_o \frac{\partial}{\partial t})\psi_L(x, t) = -m \gamma_2 \cdot \psi_L^*(x, t) \quad (7)$$

$$(\vec{\gamma} \cdot \vec{\nabla} - \gamma_o \frac{\partial}{\partial t})\psi'_L(x, t) = +m \gamma_2 \cdot \psi_L^*(x, t) \quad (8)$$

where we have taken the usual charge conjugation matrix C to be $\gamma_2 \gamma_4$ and the γ_μ matrices are all hermitian, with $\mu = 1, \dots, 4$ and $\gamma_4 \equiv -i\gamma_o$.

From this coupled set of Majorana equations, it is easy to show directly that the fields have *space-like* mass-squared. Note that if the sign in eq.(8) had been opposite, the coupled set of Majorana equations would have led to a normal *time-like* mass-squared.

The equations arise in the Lagrangian

$$L = -\bar{\psi}_L \gamma \cdot \partial \psi_L + \bar{\psi}'_L \gamma \cdot \partial \psi'_L - m \cdot \bar{\psi}_L C \bar{\psi}'_L^T - m \cdot \bar{\psi}'_L C \psi_L \quad (9)$$

As far as the Standard Model weak interactions are concerned, it is the ψ_L flavor field that participates in the charged-current and neutral-current interactions. The ψ'_L field does not. It is a sterile ghost field.

As we shall show below, this negative metric of the ψ' does not affect the unitarity of the scattering matrix element between the physical *in* and *out* vacua. The sterile ghost modes condense with the ψ_L modes to form the nilpotent physical *in* and *out* vacua.

3 Solving equations of motion

We may solve the field theory of this coupled set of Majorana field equations by using the usual expansions for the field operators in terms of the time-dependent Heisenberg annihilation and creation operators

$$\psi_L(x, t) = \frac{1}{\sqrt{V}} \sum_k \left\{ a_{k,L} \begin{pmatrix} \chi_\ell \\ 0 \end{pmatrix} + b_{-k,R}^\dagger \begin{pmatrix} \chi_r \\ 0 \end{pmatrix} \right\} e^{i\vec{k} \cdot \vec{x}} \quad (10)$$

$$\psi'_L(x, t) = \frac{1}{\sqrt{V}} \sum_k \left\{ a'_{k,L} \begin{pmatrix} \chi_\ell \\ 0 \end{pmatrix} + b'^\dagger_{-k,R} \begin{pmatrix} \chi_r \\ 0 \end{pmatrix} \right\} e^{i\vec{k} \cdot \vec{x}} \quad (11)$$

where $\chi_{\ell,r}$ are two-component helicity spinors satisfying the relation

$$(\pm \vec{\sigma} \cdot \vec{k} + k) \chi_{\ell,r} = 0 \quad (12)$$

with the properties

$$\chi_\ell(-\vec{k}) = +\eta_k^* \chi_r(\vec{k}) \quad (13)$$

$$\chi_r(-\vec{k}) = -\eta_k \chi_\ell(\vec{k}) \quad (14)$$

$$i\sigma_2 \chi_\ell^*(-\vec{k}) = -\eta_k \chi_\ell(\vec{k}) \quad (15)$$

$$i\sigma_2 \chi_r^*(-\vec{k}) = -\eta_k^* \chi_r(\vec{k}) \quad (16)$$

and η_k is the usual phase factor with the property that it is odd under parity $\vec{k} \rightarrow -\vec{k}$.

$$\eta_k = \frac{k_x + ik_y}{\sqrt{k^2 - k_z^2}} = -\eta_{-k} \quad (17)$$

The field equations (eq.(7,8)) lead to equations of motion for the Heisenberg operators:²

$$-i \dot{a} = -k a + m \eta a'^\dagger \quad (18)$$

$$-i \dot{a}' = -k a' + m \eta a^\dagger \quad (19)$$

$$-i \dot{b}^\dagger = +k b^\dagger + m \eta^* b' \quad (20)$$

$$-i \dot{b}'^\dagger = +k b'^\dagger + m \eta^* b \quad (21)$$

The solutions to these equations depend on whether the momentum, k , is greater or less than m . For convenience of notation, we shall reserve p for momenta greater than m , and q for momenta less than m . For $p > m$, we have ($\omega \equiv \sqrt{p^2 - m^2}$)

$$a_{p,L}(t) = \sqrt{\frac{p+\omega}{2\omega}} A_{p,L} e^{-i\omega t} + \sqrt{\frac{p-\omega}{2\omega}} \eta_p A'_{-p,L}^\dagger e^{+i\omega t} \quad (22)$$

$$a'_{-p,L}(t) = \sqrt{\frac{p+\omega}{2\omega}} A'_{-p,L} e^{-i\omega t} + \sqrt{\frac{p-\omega}{2\omega}} \eta_p A_{p,L}^\dagger e^{+i\omega t} \quad (23)$$

$$b_{-p,R}^\dagger(t) = \sqrt{\frac{p+\omega}{2\omega}} B_{-p,R}^\dagger e^{+i\omega t} - \sqrt{\frac{p-\omega}{2\omega}} \eta_p^* B'_{p,R} e^{-i\omega t} \quad (24)$$

$$b'_{p,R}(t) = \sqrt{\frac{p+\omega}{2\omega}} B'_{p,R} e^{-i\omega t} - \sqrt{\frac{p-\omega}{2\omega}} \eta_p B_{-p,R}^\dagger e^{+i\omega t} \quad (25)$$

while, for $q < m$, we have the complex unstable modes ($q \equiv m \cos \chi$, $\kappa \equiv \sqrt{m^2 - q^2} = m \sin \chi$)

$$a_{q,L}(t) = \frac{1}{\sqrt{2i \sin \chi}} \left(e^{+i\chi/2} A_{q,L} e^{+\kappa t} + e^{-i\chi/2} \eta_q A'_{-q,L}^\dagger e^{-\kappa t} \right) \quad (26)$$

$$a'_{-q,L}(t) = \frac{\sqrt{i}}{\sqrt{2 \sin \chi}} \left(e^{+i\chi/2} \eta_q A_{q,L}^\dagger e^{+\kappa t} + e^{-i\chi/2} A'_{-q,L} e^{-\kappa t} \right) \quad (27)$$

$$b_{-q,R}^\dagger(t) = \frac{\sqrt{i}}{\sqrt{2 \sin \chi}} \left(e^{-i\chi/2} B_{-q,R}^\dagger e^{+\kappa t} - e^{+i\chi/2} \eta_q^* B'_{q,R} e^{-\kappa t} \right) \quad (28)$$

$$b'_{q,R}(t) = \frac{\sqrt{i}}{\sqrt{2 \sin \chi}} \left(e^{-i\chi/2} B'_{q,R} e^{-\kappa t} - e^{+i\chi/2} \eta_q B_{-q,R}^\dagger e^{+\kappa t} \right) \quad (29)$$

where A, A', B, B' are *time-independent* Schrödinger operators.

²hereinafter for brevity of notation we shall write $a \equiv a_{k,L}$, $a' \equiv a'_{-k,L}$, $b \equiv b_{-k,R}$, $b' \equiv b'_{k,R}$, $\eta \equiv \eta_k$

4 Hamiltonian

In terms of the time-dependent Heisenberg operators a, a', b, b' , the Hamiltonian takes the form

$$H_o = \sum_k k (a^\dagger a - a'^\dagger a' + b^\dagger b - b'^\dagger b') \quad (30)$$

$$H_1 = -m \sum_k (a^\dagger \eta a'^\dagger + a' \eta^* a - b^\dagger \eta b'^\dagger - b' \eta^* b) \quad (31)$$

and we may directly verify that on account of the equations of motion (eq.(18-21)) we have

$$\frac{d}{dt} H = 0 \quad (32)$$

In terms of the time-independent Schrödinger operators, A, A', B, B' , the normal-ordered Hamiltonian takes the diagonal form

$$\begin{aligned} H &\equiv H_o + H_1 - \langle \Phi_{in} | H | \Phi_{in} \rangle \\ &= \sum_{p>m} \left(\omega A_{p,L}^\dagger A_{p,L} - \omega A'_{-p,L}^\dagger A'_{-p,L} \right. \\ &\quad \left. + \omega B_{-p,R}^\dagger B_{-p,R} - \omega B'_{p,R}^\dagger B'_{p,R} \right) \\ &+ \sum_{q < m} \left(-\kappa A_{q,L}^\dagger \eta_q A_{-q,L}^\dagger - \kappa A'_{-q,L} \eta_q^* A_{q,L} + q \right. \\ &\quad \left. + \kappa B_{-q,R}^\dagger \eta_q B'_{q,R} + \kappa B'_{q,R} \eta_q^* B_{-q,R} + q \right) \end{aligned} \quad (33)$$

5 Quasi-Particle Anti-Commutation Rules

The Heisenberg operators satisfy the expected equal-time anti-commutation rules for all \vec{k}, \vec{k}'

$$\left. \begin{aligned} \{a_{k,L}, a_{k',L}\} &= \{a'_{-k,L}, a'_{-k',L}\} = 0 \\ \{a_{k,L}, a'_{-k',L}\} &= \{a_{k,L}, a'^\dagger_{-k',L}\} = 0 \\ \{a_{k,L}, a^\dagger_{k',L}\} &= -\{a'_{k,L}, a'^\dagger_{-k',L}\} = \delta_{\vec{k},\vec{k}'} \end{aligned} \right\} \quad (34)$$

and likewise for the anti-particle operators.

Expressed in terms of the quasi-particle operators, however, the anti-commutation rules are different depending on whether k is greater than or less than m . For $p > m$ we have³

$$\left. \begin{aligned} \{A_{p,L}, A_{p',L}^\dagger\} &= -\{A'_{p,L}, A'^\dagger_{p',L}\} = \delta_{\vec{p},\vec{p}'} \\ \{A_{p,L}, A_{p',L}\} &= \{A'_{-p,L}, A'_{-p',L}\} = 0 \\ \{A_{p,L}, A'_{-p',L}\} &= \{A_{p,L}, A'^\dagger_{-p',L}\} = 0 \end{aligned} \right\} \quad (35)$$

while for $q < m$, the quasi-particle operators obey the unusual nilpotent relations

$$\left. \begin{aligned} \{A_{q,L}, A_{q',L}^\dagger\} &= \{A'_{-q,L}, A'^\dagger_{-q',L}\} = 0 \\ \{A_{q,L}, A_{q',L}\} &= \{A'_{-q,L}, A'_{-q',L}\} = 0 \\ \eta_q^* \{A_{q,L}, A'_{-q',L}\} &= -\eta_q \{A_{q,L}^\dagger, A'^\dagger_{-q',L}\} = -i\delta_{\vec{q},\vec{q}'} \end{aligned} \right\} \quad (36)$$

³ The anti-commutation rules for the anti-particle B and B' operators may be obtained by replacing everywhere $A_{k,L}$ and $A'_{-k,L}$ by $B_{k,R}$ and $B'_{-k,R}$ respectively.

With these commutation rules, we obtain for $p > m$ the usual operator equation of motion for the Schrödinger operators, A, A' ,

$$\left. \begin{aligned} [H, A_{p,L}] &= -\omega A_{p,L} \\ [H, A'_{-p,L}] &= -\omega A'_{-p,L} \end{aligned} \right\} (p > m) \quad (37)$$

etc. For $q < m$, however, both the $A_{q,L}$ and its conjugate, $A'_{q,L}$, carry the *same* imaginary energy

$$\left. \begin{aligned} [H, A_{q,L}] &= -i\kappa A_{q,L} \\ [H, A'_{q,L}] &= -i\kappa A'_{q,L} \\ [H, A'_{-q,L}] &= +i\kappa A'_{-q,L} \\ [H, A'_{-q,L}] &= +i\kappa A'_{-q,L} \end{aligned} \right\} (q < m) \quad (38)$$

and the vacuum carries a complex zero-point energy. This is not a problem however as the $q < m$ vacuum is nilpotent.

6 Structure of the Vacuum

Since the Hamiltonian is diagonal in k , we may consider the Hilbert space for each momentum component and study the property of the states in each subspace. For convenience, we separate the Hamiltonian in eq.(33) into the two components, H_p , superluminal Hamiltonian with $p > m$, and H_q for complex modes with $q < m$.

The physical vacuum is then given by a product of the separate vacua for each momentum component k , with the further subdivision into the a and b parts, for the particle and antiparticle sectors.

$$|\Phi_{in}\rangle = \prod_{p>m} |\Phi_p^a\rangle \otimes |\Phi_p^b\rangle \otimes \prod_{q<m} |\Phi_{q;in}^a\rangle \otimes |\Phi_{q;in}^b\rangle \quad (39)$$

For $p > m$ Hilbert space, in the particle subspace, a , the spectrum of orthogonal states may be enumerated:

	State	Norm	Energy
$a_{p,L}^\dagger 0\rangle =$	$ \Phi_p^a \rangle$	+1	0
$a_{-p,L}^\dagger 0\rangle =$	$ \Phi_p^a \rangle$	+1	ω
$A'_{-p,L}^\dagger 0\rangle =$	$ \Phi_p^a \rangle$	-1	ω
$A'_{-p,L}^\dagger A_{p,L}^\dagger 0\rangle =$	$ \Phi_p^a \rangle$	-1	2ω

where the vacuum ground state is as usual annihilated by the quasi-particle operators, A, A' . The states are all mutually orthogonal. Note that the pair of (massless) degenerate states are also given in terms of the free field operators acting on the free field vacuum, $|0\rangle$.

The complex Hamiltonian, H_q , has an unusual spectrum of states.

	State	Norm	Energy
$a_{q,L}^\dagger 0\rangle =$	$ \Phi_{q;in}^a \rangle$	0	$(q + i\kappa)$
$\eta a'_{-q,L}^\dagger 0\rangle =$	$ \Phi_{q;in}^a \rangle$	+1	q
$ \Phi_{q,out}^a\rangle \equiv A_{q,L}^\dagger A_{q,L}^\dagger 0\rangle =$	$ \Phi_{q;in}^a \rangle$	-1	q
		0	$(q - i\kappa)$

Here, the pair of degenerate states are mutually orthogonal and have vanishing inner product with both the *in* and *out* vacua. They are again related to the free field operators acting on the free field vacuum, $|0\rangle$.

In this spectrum, the pair of complex energy states are nilpotent. They obey the inner product

$$\langle \Phi_{q,out}^a | \Phi_{q,in}^a \rangle = 1 \quad (40)$$

They turn out to be the *in* and *out* scattering vacuum states. It is noteworthy that, unlike the usual quasi-particle vacuum, the *in* vacuum is annihilated by *both* A' and A'^\dagger operators, while the *out* vacuum is annihilated by A and A^\dagger operators.

$$\left. \begin{aligned} A'_{-q,L} |\Phi_q^a;in\rangle &= A'^\dagger_{-q,L} |\Phi_q^a;in\rangle = 0 \\ A_{q,L} |\Phi_q^a;out\rangle &= A^\dagger_{q,L} |\Phi_q^a;out\rangle = 0 \end{aligned} \right\} q < m \quad (41)$$

The physical interpretation of the complex vacua becomes clear when we consider their relation to the interaction picture free field vacuum, $|0\rangle$. Let $U(t_2, t_1)$ denote the time evolution operator that takes any interaction picture state vector from time t_1 to t_2 ,

$$U(t_2, t_1) = e^{-iH(t_2-t_1)} e^{+iH_o(t_2-t_1)} \quad (42)$$

and consider its action on the q subspace of the Hilbert space in the particle sector a . We find that the *in* vacuum is related to the time evolved state from $t = -\infty$ to $t = 0$, while *out* vacuum is related to the time evolved state from $t = 0$ to $t = \infty$:

$$|\Phi_q^a;in\rangle = \lim_{T' \rightarrow \infty} \frac{U(0, -T') |0\rangle}{e^{+\kappa T' - iqT'} \langle \Phi_q^a;out | 0 \rangle} \quad (43)$$

$$|\Phi_q^a;out\rangle = \lim_{T \rightarrow \infty} \frac{U^\dagger(T, 0) |0\rangle}{e^{+\kappa T + iqT} \langle \Phi_q^a;in | 0 \rangle} \quad (44)$$

(45)

The scale factors in the denominator are related to the free field vacuum expectation value of the complete time evolution $U(\infty, -\infty)$ (See below). In deriving this result, we have used the completeness relation for the subspace

$$\left. \begin{aligned} & \left\{ |\Phi_q^a;in\rangle \langle \Phi_q^a;out| \right. \\ & + \frac{i}{2} \eta^* A_{q,L} |\Phi_q^a;in\rangle \langle \Phi_q^a;out| A'_{-q,L} - \frac{i}{2} \eta A'^\dagger_{-q,L} |\Phi_q^a;out\rangle \langle \Phi_q^a;in| A^\dagger_{q,L} \\ & \left. - \frac{i}{2} \eta A^\dagger_{q,L} |\Phi_q^a;in\rangle \langle \Phi_q^a;out| A'^\dagger_{-q,L} + \frac{i}{2} \eta^* A'_{-q,L} |\Phi_q^a;out\rangle \langle \Phi_q^a;in| A_{q,L} \right\} = I_q^a \end{aligned} \right\} \quad (46)$$

as well as the identities

$$A'_{-q,L} |\Phi_q^a;out\rangle = -i\eta A^\dagger_{q,L} |\Phi_q^a;in\rangle \quad (47)$$

$$A'^\dagger_{-q,L} |\Phi_q^a;out\rangle = -i\eta^* A_{q,L} |\Phi_q^a;in\rangle \quad (48)$$

The physical vacuum state is a combined product eigenstate of both H_p and H_q

$$\begin{aligned} |\Phi_{in}\rangle &= \prod_{p>m} \left\{ \left(\cosh \theta_p + \eta_p \sinh \theta_p a_{p,L}^\dagger a'^\dagger_{-p,L} \right) \left(\cosh \theta_p - \eta_p \sinh \theta_p b_{-p,R}^\dagger b'^\dagger_{p,R} \right) \right\} \\ &\quad \prod_{q<m} \left\{ \frac{i e^{-i\chi}}{2 \sin \chi} \left(1 + e^{+i\chi} \eta_q a_{q,L}^\dagger a'^\dagger_{-q,L} \right) \left(1 - e^{+i\chi} \eta_q b_{-q,R}^\dagger b'^\dagger_{q,R} \right) \right\} |0\rangle \end{aligned} \quad (49)$$

where

$$\cosh \theta_p = \sqrt{\frac{p + \omega}{2\omega}} \quad (50)$$

Eq.(49) shows a Nambu-Jona-Lasinio[5] type condensation of the Majorana neutrinos in the physical vacuum. There are crucial differences, however. The pairing here is with ψ_L and ψ'_L fields of the same

handedness, as compared with the pairing of quarks and antiquarks in NJL. Furthermore, the vacuum supports both the tachyonic (superluminal) quasi-particle modes (for $p > m$), as well as the unstable transients $q < m$.

The physical S-matrix element taken between the complete physical *in* and *out* vacua takes the form ($x_o > y_o$)

$$\langle \Phi_{out} | \psi_L(\vec{x}, x_o) \bar{\psi}_L(\vec{y}, y_o) | \Phi_{in} \rangle = \frac{(0|U(\infty, x_o)\psi_L(\vec{x}, x_o)\bar{\psi}_L(\vec{y}, y_o)U(y_o, -\infty)|0)}{(0|U(\infty, -\infty)|0)} \quad (51)$$

so that the relation with the interaction picture becomes manifest.

For the physical S-matrix, taken with respect to the *in* and *out* vacua, the non-vanishing matrix elements are of the type

$$\begin{aligned} \langle \Phi_{out} | A_{p,L} A_{p',L}^\dagger | \Phi_{in} \rangle &= - \langle \Phi_{in} | A'_{-p,L} A'^\dagger_{-p',L} | \Phi_{out} \rangle = +\delta_{p,p'} \quad (p, p' > m) \\ \langle \Phi_{out} | A_{-q,L}^\dagger A_{q',L}^\dagger | \Phi_{in} \rangle &= \langle \Phi_{in} | A_{q,L}^\dagger A'^\dagger_{-q',L} | \Phi_{out} \rangle = i\eta_q^* \delta_{q,q'} \quad (q, q' < m) \end{aligned} \quad (52)$$

As a result, in the physical matrix elements of the time-ordered product $T(\psi_L(x)\bar{\psi}_L(y))$ between *in* and *out* vacua, the exponential runaway modes decouple, leaving behind the only transients $e^{-\kappa|x_o-y_o|}$. Co-incidentally, this decoupling of the runaway modes restores the translational invariance of the time-ordered products taken between the *in* and *out* states.

These features are important outcome of our coupled Majorana field equations. In addition to the quasi-particle superluminal modes, the proper quantization of the tachyons reveals the presence of complex transients $e^{-\kappa|x_o-y_o|}$. Earlier work in condensed matter physics⁴ suggests that we interpret this transient observed at x_o as being associated with the population inversion at the earlier time y_o .

7 Micro-Causality

The neutrino field, ψ_L , satisfies the normal causality relations with respect to the S-matrix element between the *in* and *out* vacua.

$$\begin{aligned} \langle \Phi_{out} | \{\psi_L(x), \bar{\psi}_L(y)\} | \Phi_{in} \rangle &= 0 \\ \langle \Phi_{out} | \{\bar{\psi}'_L(x), \psi_L(y)\} | \Phi_{in} \rangle &= 0 \end{aligned} \quad \left. \right\} \quad (x-y)^2 \text{ spacelike} \quad (53)$$

The equal time relations of the physical ψ_L field remain valid with respect to the S-matrix elements

$$\langle \Phi_{out} | \{\psi_L(\vec{x}, 0), \psi_L^\dagger(\vec{y}, 0)\} | \Phi_{in} \rangle = \delta(\vec{x} - \vec{y}) \quad (54)$$

The time-ordered Green function for the physical ψ_L field is given by

$$\langle \Phi_{out} | T(\psi_L(x)\bar{\psi}_L(y)) | \Phi_{in} \rangle = \int \frac{d^4k}{(2\pi)^4} \frac{-\vec{\gamma} \cdot \vec{k} - \gamma_o \cdot k_o}{\vec{k}^2 - k_o^2 - m^2 - i\epsilon} e^{i\vec{k} \cdot (\vec{x} - \vec{y}) - ik_o(x_o - y_o)} \quad (55)$$

and by closing the contour in the lower half k_o plane for $x_o > y_o$ we recover the quasi-particle contributions from $k > m$, as well as the transient mode contributions from $k < m$.

⁴In condensed matter physics, the work of Chiao and others [13] have shown that superluminal propagation occurs in a medium with inverted population. The inverted population lead to unstable modes that are necessary for causality (see Aharonov and co-workers[14]).

8 Time Evolved State

In this model, the negative metric ψ'_L is a sterile field, it is not created by the usual weak and electromagnetic interactions. Its role is to condense with the physical ψ_L field to form the *in* and *out* vacua. In the physical S-matrix elements that are to be taken between *in* and *out* vacua, only ψ_L^\dagger and ψ_L operators are present to create and annihilate physical neutrinos.

The physical neutrino and anti-neutrino states with $p > m$ created at $t = 0$ from the in-vacuum is represented by⁵

$$|\nu_L, \vec{p}, 0\rangle = \int \frac{d^3x}{\sqrt{V}} \psi_L^\dagger(\vec{x}, 0) u_L |\Phi_{in}\rangle e^{i\vec{p}\cdot\vec{x}} \quad (56)$$

$$|\bar{\nu}_R, \vec{p}, 0\rangle = \int \frac{d^3x}{\sqrt{V}} \tilde{\psi}_L(\vec{x}, 0) C v_R |\Phi_{in}\rangle e^{i\vec{p}\cdot\vec{x}} \quad (57)$$

where

$$\begin{aligned} u_L &= \begin{pmatrix} c\chi_\ell + s\chi_r \\ 0 \end{pmatrix}, & u_R &= \begin{pmatrix} 0 \\ c\chi_r - s\chi_\ell \end{pmatrix} \\ v_R &= \begin{pmatrix} c\chi_r - s\chi_\ell \\ 0 \end{pmatrix}, & v_L &= \begin{pmatrix} c\chi_\ell + s\chi_r \\ 0 \end{pmatrix} \end{aligned} \quad (58)$$

are the positive and negative energy spinor solutions of the equation

$$i(\vec{\gamma} \cdot \vec{p} - \gamma_0 \omega) u_{L,R} = m\gamma_5 u_{R,L} \quad (59)$$

$$i(\vec{\gamma} \cdot \vec{p} + \gamma_0 \omega) v_{L,R} = m\gamma_5 v_{R,L} \quad (60)$$

$$(61)$$

and we have used the abbreviation $c \equiv \sqrt{\frac{p+\omega}{2p}}$ and $s \equiv \sqrt{\frac{p-\omega}{2p}}$, with $c^2 + s^2 = 1$.

$$|\nu_L, \vec{p}; t\rangle = e^{-iHt} |\nu_L, \vec{p}, 0\rangle \quad (62)$$

$$= e^{-i\omega t} \left(c \cosh \theta_p A_{p,L}^\dagger - s \sinh \theta_p \eta_p B'_{p,R}^\dagger \right) |\Phi_{in}\rangle \quad (63)$$

Eq.(63) shows that *in the one-flavor case*, the physical neutrino state is actually a superposition of the physical quasi-particle mode, $A_{q,L}$, and the negative metric quasi-antiparticle mode, $B'_{q,R}$. The time evolution of this state does not show an oscillation into a right-handed neutrino state. Instead the state behaves as an eigenstate of the Hamiltonian, with the usual (superluminal) time dependence. There is thus *no neutrino deficit* as it propagates in the ether. The state remains of positive unit norm as time evolves.

In establishing the time evolution of this Schrödinger state vector, we note a useful set of identities with respect to the states created by the fields

$$\int d^3x \bar{\psi}_L(\vec{x}, t) |\Phi_{in}\rangle \gamma_5 u_R e^{ip\cdot x} = - \int d^3x \tilde{\psi}'_L(\vec{x}, t) |\Phi_{in}\rangle C u_L e^{ip\cdot x} \quad (64)$$

$$\int d^3x \bar{\psi}'_L(\vec{x}, t) |\Phi_{in}\rangle \gamma_5 u_R e^{ip\cdot x} = - \int d^3x \tilde{\psi}_L(\vec{x}, t) |\Phi_{in}\rangle C u_L e^{ip\cdot x} \quad (65)$$

⁵ Transient states with $q < m$ are exponentially damped in time after creation at $t = 0$, and do not contribute to neutrino oscillations.

9 Reduction Formulae

The scattering matrix element for the physical neutrino field may be expressed in terms of the reduction formulae. Let $|\nu_L; p >_{in}$ denote the incoming scattering state for the superluminal neutrino with space-like momentum p , then

$$|\nu_L; p >_{in} = \lim_{t \rightarrow -\infty} \int \frac{d^3x}{\sqrt{V}} \bar{\psi}_L(x) |\Phi_{in} > \gamma_4 u_L e^{ip \cdot x} \quad (66)$$

$$= \left(c \cosh \theta_p A_{p,L}^\dagger - s \sinh \theta_p \eta_p B_{p,R}^\dagger \right) |\Phi_{in} > \quad (67)$$

10 3-Flavor Phenomenology

The toy model we have considered so far consists of one left-handed flavor mixing with a sterile left-handed neutrino field. The toy model may be made realistic by having three flavors of ψ_L^α , $\alpha = (e, \mu, \tau)$ fields coupled to the sterile left-handed ψ'_L field:⁶

$$(\vec{\gamma} \cdot \vec{\nabla} - \gamma_o \frac{\partial}{\partial t}) \psi_L^\alpha = -m u_3^\alpha \gamma_2 \psi'^* \quad (68)$$

$$(\vec{\gamma} \cdot \vec{\nabla} - \gamma_o \frac{\partial}{\partial t}) \psi'_L = +m u_3^\alpha \gamma_2 \psi_L^\alpha \quad (69)$$

Here u_β^α is the unitary mixing matrix that rotates from the Standard Model flavor basis to the eigenstates of the Majorana equation: two massless left-handed neutrino fields, $\psi_L^{(1)}, \psi_L^{(2)}$, together with the tachyonic massive 4-component field made up of $\psi_L^{(3)}$ and the sterile ψ'_L , where

$$(\vec{\gamma} \cdot \vec{\nabla} - \gamma_o \frac{\partial}{\partial t}) \psi_L^{(1)} = 0 \quad (70)$$

$$(\vec{\gamma} \cdot \vec{\nabla} - \gamma_o \frac{\partial}{\partial t}) \psi_L^{(2)} = 0 \quad (71)$$

$$(\vec{\gamma} \cdot \vec{\nabla} - \gamma_o \frac{\partial}{\partial t}) \psi_L^{(3)} = -m \gamma_2 \psi'^* \quad (72)$$

$$(\vec{\gamma} \cdot \vec{\nabla} - \gamma_o \frac{\partial}{\partial t}) \psi'_L = +m \gamma_2 \psi_L^{(3)} \quad (73)$$

The time evolution of the neutrino flavor α state now is of the form

$$|\nu^\alpha; p; t > = u_1^\alpha e^{-ipt} |\nu^{(1)}; p; 0 > + u_2^\alpha e^{-ipt} |\nu^{(2)}; p; 0 > + u_3^\alpha e^{-i\omega t} |\nu^{(3)}; p; 0 > \quad (74)$$

and the oscillation into the neutrino flavor β state is found by looking at the overlap

$$\langle \nu^\beta; p; 0 | \nu^\alpha; p; t > = (u_\beta^1 u_1^\alpha + u_\beta^2 u_2^\alpha) e^{-ipt} + u_\beta^3 u_3^\alpha e^{-i\omega t} \quad (75)$$

$$= e^{-ipt} \left(\delta_\beta^\alpha + 2i u_\beta^3 u_3^\alpha e^{i(p-\omega)t/2} \sin \frac{(p-\omega)t}{2} \right) \quad (76)$$

$$\approx e^{-ipt} \left(\delta_\beta^\alpha + 2i u_\beta^3 u_3^\alpha e^{im^2 t/4p} \sin \frac{m^2 t}{4p} \right) \quad (77)$$

Eq.(77) is indistinguishable from the usual Dirac flavor oscillation formula. Even though the physical neutrino $\nu^{(3)}$ has a tachyonic mass, the neutrino oscillation rates take the same form as in the temporal mass case

$$P_{\alpha \rightarrow \alpha} = \left(1 - 2|u_3^\alpha|^2 \left(\sin \frac{m^2 t}{4} \right)^2 \right)^2 + |u_3^\alpha|^4 \left(\sin \frac{m^2 t}{2} \right)^2 \quad (78)$$

⁶ For simplicity, we have not considered the more general case with three different tachyonic masses. This may be obtained by coupling the three flavor neutrinos to three sterile neutrinos.

$$P_{\alpha \rightarrow \beta} = 4|u_{\beta}^3|^2 |u_3^{\alpha}|^2 \left(\sin \frac{m^2 t}{4p} \right)^2 \quad (79)$$

11 Conclusion

In this paper, we have studied the canonical quantization of an interacting tachyonic majorana field theory. We have shown how micro-causality is preserved in the physical matrix elements taken between the *in* and *out* states. In the single flavor case, the physical neutrino state evolves as a tachyonic mass state, with no neutrino deficit. In the 3-flavor case, there can be neutrino oscillation due to flavor mixing. The phenomenology is indistinguishable from the usual timelike oscillations.

Studies of the effect of the tachyonic mass on the β -decay spectrum are underway, and will be reported in a future publication.

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